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Analysis of Gear Strength by Static and Dynamic Finite Element Methods

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Abstract: As key factor of numerical simulation for gear strength, namely determination of effective loads between meshing teeth profiles, their numerical calculation principles of static and dynamic finite element methods are analyzed according to the loads on gear teeth. The static analysis of gear strength can be classified to effective statics and static contact analysis, and the numerical simulation method for face distribution as the rule of Hertz contact is advanced. The differences between static and dynamic contact and their influences in results are researched. Compared with the empirical formulas, the analytical results about bearing capacity of gear drive under different effective loads show: (1) As the sequence of concentrated force, linear distribution force, face distribution force as the rule of Hertz contact, static contact analysis, the accuracy and reliability of calculating results under different effective loads are improved, especially the results from face distribution and static contact being very close. (2) Dynamic contact analysis can be used to simulated the effect well. The above studies could be referenced to calculation in bearing capacity of gear drive.

1 Introduction

The calculation of the tooth carrying capacity is very important for gear transmission system. Comparing to other numerical methods, the finite element method (FEM) can handle complex load cases and boundary conditions and obtain force field distribution and teeth deformation.

For teeth bending strength analysis, Chabert [1] used the FEM to study the gear teeth stress and deformation. The effects of gear shape, boundary condition and mesh cell size are considered by Von Eiff [2]. Filliz [3] investigated effect of the design parameters on the gear teeth stress based on three static loads. Baund [4] has investigated static/dynamic tooth loading in geared rotor systems and studied the influence of external and parametric excitations. The effect of tooth asymmetry and tooth root shape on the root stresses are studied by performing an experiment [5].Then, based on the minimum elastic potential energy criterion, a model of non-uniform load distribution along the line of contact is applied to determine the critical tooth-root stress [6].

For teeth contact strength analysis, Corry [7] calculated the gear's elastic deformation and contact stress. Refaat [8] calculated the contact stress and gear tooth root stress by combing with variational inequalities and FEM. Li [9] presented a method that combined the mathematical programming method with three-dimensional FEM to study the loaded tooth contact. A finite element method for 3D contact/impact problem is proposed according to the derivation of a flexibility matrix equation in the contact region [10]. Based on a 2D and 3D finite element model, Hwang [11] presented a contact stress analysis for a pair of mating gears during rotation which concentrates on the change in the contact stress generated in meshing gear teeth. José I. Pedrero [12] studied the contact stress for the spur and helical gears with and without undercut condition.

Above all, the previous references [1-12] are devoted to analyze the gear stress and deformation under static load, dynamic load and impact load condition, but the effect of loads on the stress results is still need to comprehensive analysis. The computing model and effective load play a decisive role in the solution accuracy especially for the complex variable curvature conjugate gear surfaces contact problem. The main motivation of the present works is to analyze the gear stress under static load and dynamic load condition based on the accurate analytical model.

2 Principle of Static and dynamic FEM in Gear Strength Analysis

2.1 Principle of static FEM in gear strength analysis

The static analysis of gear transmission system, especially at a low speed condition is a small deformation problem. The elasto-static basic control equations are

$$\begin{cases} Equilibrium\ equations: & \sigma_{ij,j} + \overline{f_i} = 0 \\ Geometric\quad equations: & \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \\ Constitutive\ equations: & \sigma_{ij} = 2G\varepsilon_{ij} + \lambda\delta_{ij}\delta_{kl} \end{cases} \tag{1}$$

Here, $\sigma_{ij,j}$ is the deviation of stress tensor σ_{ij}, $\overline{f_i}$ is the volume force tensor, ε_{ij} is the strain tensor, $u_{i,j}$ and $u_{j,i}$ are the deviation of displacement tensor, G and λ are Lame constant, δ_{ij} and δ_{kl} are Kronecher function. And the force and displacement boundary are

$$T_i = \overline{T_i}, u_i = \overline{u_i} \tag{2}$$

Here, T_i is the boundary of internal force tensor of units area, $\overline{T_i}$ is the applied force tensor of boundary, u_i is the displacement tensor and $\overline{u_i}$ is the known boundary displacement tensor.

The basic idea using FEM to analyze the gear strength is the discretization of gear body, namely the use of the principle of minimum potential energy to create a finite element discretization equation using the FEM, as

$$\Pi_p = \int_V \left(\frac{1}{2} D_{ijkl}\varepsilon_{ij}\varepsilon_{kl} - \overline{f_i}u_i \right) dV - \int_{S_\sigma} \overline{T_i}u_i dS \tag{3}$$

and solving equation are

$$\mathbf{Ku} = \mathbf{P} \tag{4}$$

$$\mathbf{P} = \sum_e \mathbf{G}^{\mathrm{T}}\mathbf{P}^e \tag{5}$$

Here, D_{ijkl} is the elastic constant, ε_{kl} is strain tensor, $\mathbf{K}$ is the structural stiffness matrix, $\mathbf{u}$ node displacement vector, $\mathbf{P}$ is the node load vector, $\mathbf{G}^{\mathrm{T}}$ is the transformation matrix and $\mathbf{P}^e$ is the equivalent load vector.

2.2 Principle of dynamic FEM in gear strength analysis

Gear dynamic contact is a strong nonlinear behavior, which is included normal contact, and transit impact due to gear error, deformation, backlash and fluctuation velocity. The dynamic analysis results will reveal the stress distribution, deformation and their change laws.

The governing equation of dynamic analysis is similar to static analysis. The equilibrium equation can be written as

$$\sigma_{ij,j} + f_i - \rho u_{i,tt} - \mu u_{i,t} = 0 \tag{6}$$

Here, ρ is density of mass, μ is damping coefficient, $u_{i,tt}$ and $u_{i,t}$ are the acceleration and velocity at direction *i*. Based on the Hamilton variation equation, the gear dynamic contact problem can be solved numerically,

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{Q}(t) \tag{7}$$

Here, $\ddot{\mathbf{u}}(t)$, $\dot{\mathbf{u}}(t)$ and $\mathbf{u}(t)$ are the acceleration vector, velocity vector and displacement vector of node, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are mass matrix, damping matrix and stiffness matrix, $\mathbf{Q}(t)$ is load vector.

Neglecting the effect of damping, the variation equation can be solved by the Lagrange multiplier method as

$$\mathbf{M}^{t+\Delta t}\ddot{\mathbf{u}} + ({}_0^t\mathbf{K}_L + {}_0^t\mathbf{K}_{NL})\mathbf{u} - {}^{t+\Delta t}\mathbf{Q}_C = {}^{t+\Delta t}\mathbf{Q}_L - {}_0^t\mathbf{F} \tag{8}$$

Here, ${}^{t+\Delta t}\ddot{\mathbf{u}}$ is the node acceleration vector at time $t+\Delta t$, ${}_0^t\mathbf{K}_{NL}$ is initial stress matrix, ${}^{t+\Delta t}\mathbf{Q}_C$ is equivalent contact force, ${}^{t+\Delta t}\mathbf{Q}_L$ is a equivalent load vector. And,

$$ {}_0^t\mathbf{K}_L = \sum_e \int_{{}^0V} {}_0^t\mathbf{B}_L^T \, {}_0\mathbf{D} \, {}_0^t\mathbf{B}_L \, {}^0dV \tag{9}$$

$$ {}_0^t\mathbf{F} = \sum_e \int_{{}^0V} {}_0^t\mathbf{B}_L^T \, {}_0^t\hat{\mathbf{S}} \, {}^0dV \tag{10}$$

Here, ${}_0^t\mathbf{B}_L^T$ is transformation matrix of linear strain and displacement, ${}_0\mathbf{D}$ is the material constitutive matrix, ${}_0^t\hat{\mathbf{S}}^0$ is the second kinds of Piola-Kirchhoff stress vector. Additional, the explicit central difference method and the implicit Newmark method is widely used in applications. As for the duration of gear contact is short and the two steps explicit central difference method as

$$ {}^{t+\Delta t}\mathbf{u} = {}^t\mathbf{u} + {}^t\dot{\mathbf{u}}\Delta t + \frac{1}{2}{}^t\ddot{\mathbf{u}}\Delta t^2 \tag{11}$$

$$ {}^{t+\Delta t}\dot{\mathbf{u}} = {}^t\dot{\mathbf{u}} + \frac{1}{2}\left({}^t\ddot{\mathbf{u}} + {}^{t+\Delta t}\ddot{\mathbf{u}}\right)\Delta t^2 \tag{12}$$

and substituting Eqs. (11) and (12) into (8), one can obtain,

$$\mathbf{M}^{t+\Delta t}\ddot{\mathbf{u}} = {}^{t+\Delta t}\mathbf{Q}_L + {}^{t+\Delta t}\mathbf{Q}_C - {}_0^{t+\Delta t}\mathbf{F} \tag{13}$$

3 Numerical results and discussion

The analytical gear model used in present paper is built by the generating principle and the method for compositely modeling by parameterization, which is manufactured by standard gear hob [13]. The elastic deformations of cutting tool and gear body are not included. As mentioned in the previous sections, two kinds of gear contact strategies, static and dynamic contact are performed in this section. The gear parameters are: gear tooth Z_1 and pinion tooth Z_2 are 30 respectively, pressure angle is 20°, module is 6mm, tooth width is 80mm, power is 5.5kW, speed is 100r/min, Young's modulus are 206GPa, shear modulus is 79.4GPa, Poisson's ratio are 0.3 and material density is 7850kg/m^3. The assembling error, gear manufacture error and gear box deformation are neglected. The gear 3D model with gear body is shown in Fig.1.

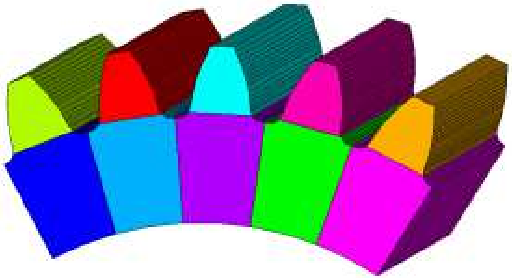
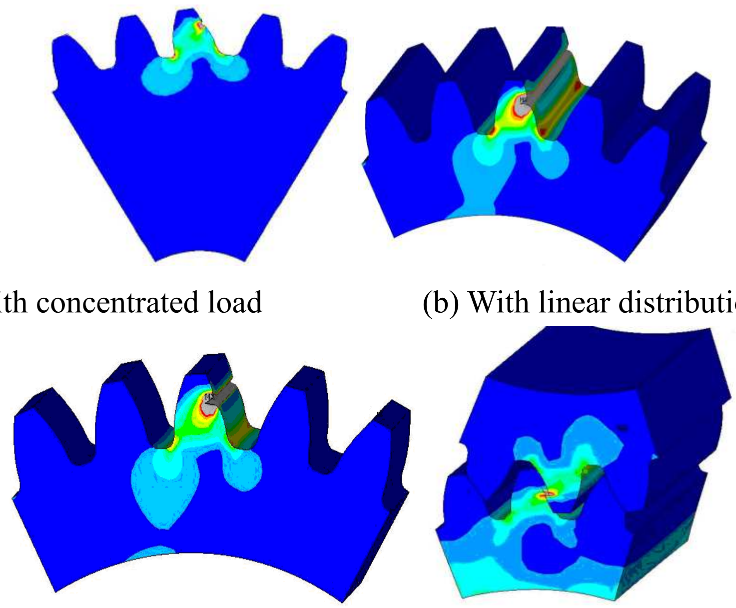


Fig. 1 Schematic view of gear

3.1 Results of gear strength by static FEM

Neglecting the friction effect, the static stress and its distribution with different load conditions, (a) concentrated load, (b) linear distribution load, (c) face distribution load and (d) Hertzian contact force, are shown in Fig.2. The Mises stress and deformation contour plots with different load conditions are similar.



(a) With concentrated load (b) With linear distribution load

(c) With face distribution load (d) With Hertzian contact force

Fig. 2 Tooth deformation and Mises stress distribution with different load conditions

The peak stress (maximum compression and tensile stress) values for the four load conditions are listed in Table 1. The results show that the peak stress under concentrated load is the largest, and other three are closed. The results are more reliable when the Hertzian contact force is adopted in the gear static analysis. To further validate the numerical results, the maximum tensile stress calculated by empirical formula or standard in Chabert[1], Niemann [14], Fillize [4] and Ref. [15] are compared as listed in Table 2. The results show that our numerical results is closed to the previous results but it different from Charbert's results. The main reasons are (1) different load boundary conditions are included and (2) different load position.

Table 1 Peak root stress with different loads in static analysis(MPa)

Root peak stress	Concentrated load	Linear distribution load	Face distribution load	Hertzian contact force
σ_{cmax}	43.563	39.069	40.858	40.683
σ_{tmax}	36.470	32.669	33.013	33.010

Table 2 Maximum tensile stress with different papers(MPa)

Maximum tensile stress	Charbert [1]	Niemann [14]	Fillize [4]	Ref. [15]
σ_{tmax}	29.629	32.968	33.663	33.306

Table 3 shows the deformations for the four kinds load conditions. It indicate that the deformations for the first two case is larger than other two about 6.7%. The deformation with Hertzian contact force is closed to the static analysis results, which show that the Hertzian contact force is more realistic. Note that the deformation in Ref. [16] is small for the contact deformation is not included.

Table 3 Deformations of Loaded point(um)

Deformation	Ref. [15]	Concentrated load	Linear distribution load	Face distribution load	Hertzian contact force
δ	2.700	3.250	3.202	3.070	3.012

3.2 Results of gear strength by dynamic FEM

The gear deformation and stress at high contact point of single tooth is shown in Fig.3, and the peak stress and deformation values are listed in Table 4 and Table 5 respectively. The results show that the dynamic analysis will appear real dynamic characteristic in gear transmission system, and the peak root stress is higher than 3.8% - 6.8%, and gear deformation is 7.6%.

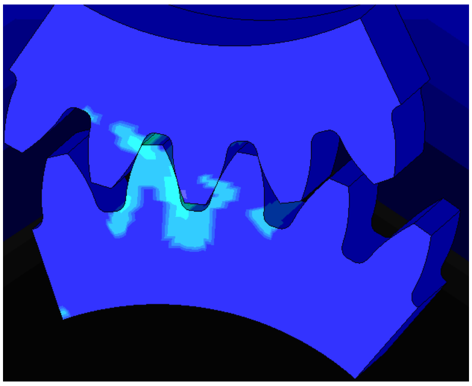


Fig. 3 Dynamic gear root stress

Table 4 Root peak stress (MPa)

Root stress	Face distribution load	Hertzian contact force	Dynamic contact force
σ_{cmax}	40.858	40.683	42.180
σ_{tmax}	33.013	33.010	35.240

Table 5 Root peak deformation (um)

Deformation	Face distribution load	Hertzian contact force	Dynamic contact force
δ	3.070	3.012	3.245

4 Conclusion

(1) The effective loads between meshing teeth profiles are classified to static loads and dynamic contact/impact. The results of numerical simulation for gear strength are closely depended on the types of the loads.

(2) For different load conditions, (a) concentrated load, (b) linear distribution load, (c) face distribution load and (d) Hertzian contact force are considered to obtain the gear tooth static deformation and stress, and compare with previous results show that the face distribution load is closed to the static analysis results.

(3) The results from dynamic FEM is more realistic than the ones from static FEM in gear strength analysis.

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