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On the design of slider-crank mechanisms. Part I: multi-phase motion generation

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Abstract

A method for designing slider-crank mechanisms to achieve multi-phase motion generation applications typically accomplished by adjustable planar four-bar motion generators is presented. The benefit of this method is twofold. First, multiple phases of prescribed rigid body positions are achievable using a mechanism with fewer moving parts than the planar four-bar mechanism. Second, the slider-crank motion generator can achieve phases of prescribed rigid body positions without any physical or automated adjustments of its moving pivots between phases. A slider path that enables the slider-crank motion generator to achieve two phases of prescribed rigid body positions is designed by using 7th order polynomials to connect the moving pivot paths of the follower link of the adjustable planar four-bar motion generator. This polynomial generates smooth radial displacement, velocity, acceleration and jerk profiles with boundary conditions that can be prescribed. The example problem in this work considers a two-phase moving pivot adjustment of a planar four-bar mechanism.

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1. Introduction

Planar four-bar mechanisms are widely used in mechanical systems and devices. Due to the planar kinematics, joint type and joint axis orientations of the planar four-bar mechanism, it can be practical to design and implement these mechanisms (compared to most four-bar spatial mechanisms). In addition, an extensive array of graphical and analytical design and analysis methods exists for planar four-bar mechanisms.

Motion generation problems in mechanism synthesis require that a rigid body be guided through a series of prescribed positions. The four-bar linkage shown in Fig. 1 can be used to produce this motion by making the rigid body as a part of its coupler link. Fig. 2 shows motion generation for the three positions in an assembly machine. An ideal motion of the coupler can only be approximated by several discrete precision positions. Since a linkage has only a finite number of significant dimensions, the designer may only prescribe a finite number of precision points. A four-bar linkage can satisfy up to five prescribed positions for the motion generation problem. However, an adjustable four-bar linkage can satisfy more than five given positions with the same

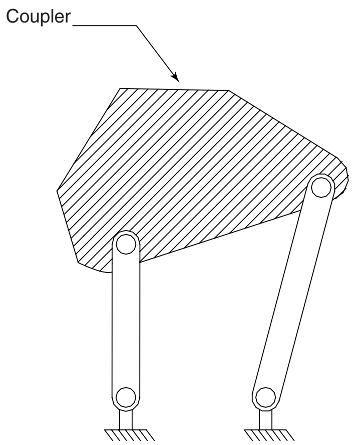

Fig. 1. Planar four-bar mechanism.

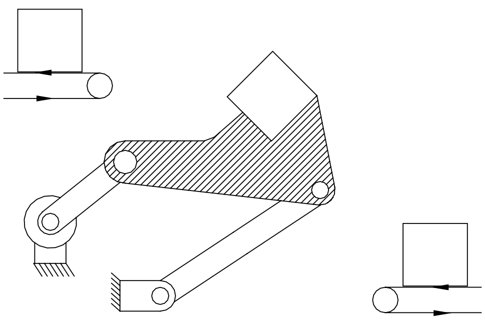

Fig. 2. Planar four-bar loading mechanism.

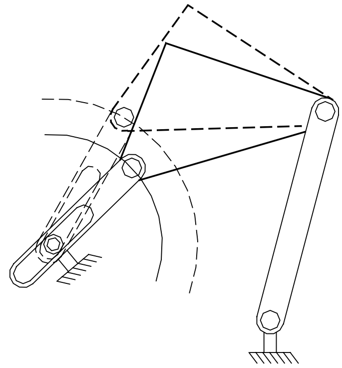

Fig. 3. Adjustable crank length.

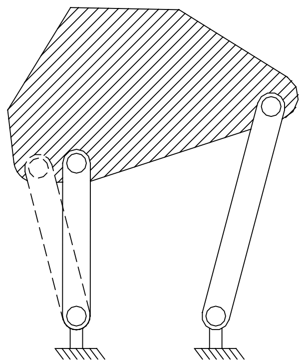

Fig. 4. Fixed crank length.

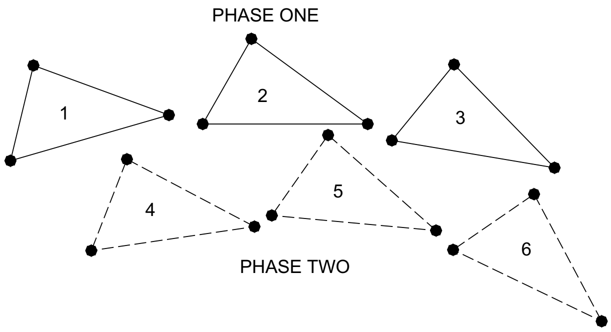

Fig. 5. Two phases of prescribed rigid body positions.

hardware. The moving pivots of a four-bar linkage can beadjusted in two different ways: with adjustable crank/follower lengths (Fig. 3) and with fixed crank/follower link adjustments (Fig. 4).

The adjustable linkages can provide solution for two phases of general plane motion (Fig. 5). If a four-bar linkage is designed to reach positions 1, 2 and 3 in phase 1, after the adjustments, the same linkage can reach three new positions 4, 5 and 6 in the second phase 2. Both phases of motion can be accomplished using the same hardware by adjusting one or more of the linkage parameters. The linkage can create the motion precisely at these positions and will approximate the motion at other positions. The more precision positions are used, the closer to the ideal motion is the actual motion of the coupler.

In the area of adjustable linkages for motion generation, published work is somewhat limited [1–19]. Previous work includes the work of Ahmad and Waldron [1] who developed a technique for synthesizing a four-bar linkage with adjustable driven fixed pivot. They solved two-phase problems with a maximum total number of five positions. Tao and Krishnamoorthy [2] developed graphical synthesis procedures to generate variable coupler curves with cusps. McGovern and Sandor [3,4] presented methods to synthesize adjustable mechanisms for function and path generation using complex variables. Funabashi et al. [5] presented general methods to design planar, spherical and spatial mechanisms which can adjust input–output relationships. Shoup [6] designed adjustable spatial slider-crank mechanism to be used as a variable displacement pump. Cheunchom and Kota [7] have presented general methods for the synthesis of adjustable mechanisms using adjustable dyads. Wilhelm [8] developed synthesis techniques for two-phase motion generation problems of adjustable four-bar linkages. Wang and Sodhi [9] developed solutions for the two-phase adjustable moving pivot problems with three positions in each of the two phases. Russell and Sodhi [10,11] recently presented methods for synthesizing adjustable three-dimensional mechanisms for multi-phase motion generation with tolerances. Using these methods, spatial RRSS mechanisms can be synthesized to achieve phases of prescribed precise rigid body positions and rigid body positions with tolerances. Recently Chang [12] presented synthesis of adjustable four-bar mechanisms generating circular arcs with specified tangential velocities.

If there is any performance-related limitation to the adjustable planar four-bar mechanism, it is that manual or automated adjustments are required to achieve all of the prescribed phases in multi-phase applications. Manual adjustments can be time consuming—especially if the adjustment procedure is involved and the mechanism adjustments must be performed frequently. Implementing automated adjustment capabilities may make the mechanism impractical from a financial standpoint—especially when operations and maintenance expenditures are considered.

For an adjustable planar four-bar motion generator that incorporates both moving pivot and link length adjustments for the follower link and only moving pivot adjustments for the crank link, an equivalent slider-crank motion generator can be designed to achieve multiple phases of prescribed rigid body positions. The benefits of the method are that multiple phases of prescribed rigid body positions are achievable using a mechanism with fewer moving parts than the planar four-bar mechanism and the slider-crank motion generator can achieve phases of prescribed rigid body positions without any physical or automated adjustments of its moving pivots between phases.

In this work, a method to design slider-crank motion generators to achieve multi-phase motion generation applications typically accomplished by adjustable planar four-bar motion generators is presented. A slider path that enables the slider-crank motion generator to achieve two phases of rigid body positions is designed by using 7th order polynomials to connect the moving pivot paths of the follower link of the adjustable planar four-bar motion generator. The radial displacement, velocity, acceleration and jerk parameters of the moving pivot of the follower link are also prescribed using the boundary conditions of these polynomials.

2. Rigid body guidance and multi-phase motion generation

The slider-crank motion generator design method presented in this work is adaptable to virtually any multi-phase motion generation method available that incorporates moving pivot adjustments with fixed and adjustable crank and follower lengths respectively. The authors [10,11] developed the multi-phase motion generation method utilized in this work.

The planar four-bar motion generator is illustrated in Fig. 6. In this work, link $\mathbf{a}_0$–$\mathbf{a}_1$ is the designated crank link and link $\mathbf{b}_0$–$\mathbf{b}_1$ is the designated follower link. Links $\mathbf{a}_0$–$\mathbf{a}_1$ and $\mathbf{b}_0$–$\mathbf{b}_1$ of the planar four-bar mechanism must satisfy the constant length condition only since its fixed and moving pivot joint axes remain parallel. Given a fixed pivot $\mathbf{b}_0$ and a moving pivot $\mathbf{b}_1$ the constant length condition in Eq. (1) [20,21] must be satisfied when synthesizing the crank and follower links of the planar four-bar mechanism.

$$(\mathbf{b}_j - \mathbf{b}_0)^{\mathrm{T}}(\mathbf{b}_j - \mathbf{b}_0) = (\mathbf{b}_1 - \mathbf{b}_0)^{\mathrm{T}}(\mathbf{b}_1 - \mathbf{b}_0) \quad j = 2, 3, \ldots, n \tag{1}$$

where

$$\mathbf{b}_0 = (b_{0x}, b_{0y}, 1) \quad \mathbf{b}_1 = (b_{1x}, b_{1y}, 1) \quad \mathbf{b}_j = [D_{ij}]\mathbf{b}_1$$

and

$$[D_{ij}] = \begin{bmatrix} p_{jx} & q_{jx} & r_{jx} \\ p_{jy} & q_{jy} & r_{jy} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_{ix} & q_{ix} & r_{ix} \\ p_{iy} & q_{iy} & r_{iy} \\ 1 & 1 & 1 \end{bmatrix}^{-1} \tag{2}$$

Eq. (1) can be rewritten as Eq. (3). In Eq. (3), the variable R represents the length of the crank or follower link.

One objective of this work is to design an equivalent slider-crank motion generator for an adjustable planar four-bar motion generator. Although the moving pivots of both the crank

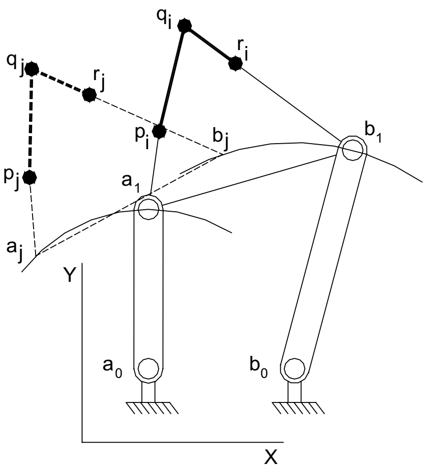

Fig. 6. The planar four-bar motion generator with rigid body points **p**, **q** and **r**.

and follower link of the planar four-bar mechanism are adjustable, only the length of the follower link will be adjusted (not the crank link). By doing this, the equivalent slider-crank motion generator to be designed will have a fixed crank link length and a slider path that accounts for the adjustment of the follower link.

$$(\mathbf{b}_j - \mathbf{b}_0)^{\mathrm{T}}(\mathbf{b}_j - \mathbf{b}_0) = R^2 \quad j = 2, 3, \ldots n \tag{3}$$

Eq. (2) is a rigid body displacement matrix. It is a derivative of the spatial rigid body displacement matrix [20,21]. Given the coordinates for a rigid body in position "i" and the subsequent "j," matrix $[D_{ij}]$ is the transformation matrix required to transform coordinates from position "i" to position "j." Variables **p**, **q** and **r** in Eq. (2) represent the position of the rigid body in two-dimensional space. Although the position of a rigid body in two-dimensional space is commonly described by a single point and a displacement angle (**p** and θ for example), the authors chose to describe the rigid body using three points for computational purposes. If the user prefers to describe the rigid body using conventional notation, the displacement matrix in Eq. (2) will be replaced with the conventional plane rigid body displacement matrix [20,21]. Since there are four variables (b_{0x}, b_{0y}, b_{1x} and b_{1y}), a maximum of five rigid body positions can be prescribed, with no arbitrary choice of parameter for one phase (see Table 1).

Points **p**, **q** and **r** should not all lie on the same line in each rigid body position. Taking this precaution prevents the rows in the rigid body displacement matrix (Eq. (2)) from becoming proportional. With proportional rows, this matrix cannot be inverted.

In Table 1, the *maximum* numbers of prescribed rigid body positions for the adjustable planar four-bar motion generator for several phases are given. The number of fixed and moving pivot coordinates for the crank and follower links determine the maximum number of rigid body positions. In the example problem in this work, an equivalent slider crank is designed to achieve a two-phase moving pivot adjustment application for an adjustable planar four-bar motion generator.

In the two-phase, adjustable moving pivot example problem in this work, the required unknowns are $\mathbf{a}_0$, $\mathbf{a}_1$, $\mathbf{a}_{1n}$, $\mathbf{b}_0$, $\mathbf{b}_1$ and $\mathbf{b}_{1n}$. The unknowns $\mathbf{a}_0$ and $\mathbf{b}_0$ represent the fixed pivots of the planar four-bar mechanism. The unknowns $\mathbf{a}_1$, $\mathbf{a}_{1n}$, $\mathbf{b}_1$ and $\mathbf{b}_{1n}$ represent the moving pivots in phase 1 and phase 2 of the planar four-bar mechanism. Since each of these unknowns has two components, there are a total of 12 variables to determine.

$$\mathbf{a}_0 = (a_{0x}, a_{0y}) \quad \mathbf{a}_1 = (a_{1x}, a_{1y}) \quad \mathbf{a}_{1n} = (a_{1nx}, a_{1ny})$$
$$\mathbf{b}_0 = (b_{0x}, b_{0y}) \quad \mathbf{b}_1 = (b_{1x}, b_{1y}) \quad \mathbf{b}_{1n} = (b_{1nx}, b_{1ny})$$

Table 1
Prescribed rigid body position and phase variations for the adjustable planar four-bar mechanism

Number of phases	Maximum number of rigid body positions	Crank or follower links	
		Number of unknowns	Number of free choices
1	5	4	0
2	8	6	0
3	11	8	0
m	$5 + 3(m - 1)$	$2 + 2m$	0

Eqs. (4)–(8), were used to calculate five of the six unknowns in $\mathbf{a}_0$, $\mathbf{a}_1$ and $\mathbf{a}_{1n}$. The variable a_{0x} and the link length R_1 are specified.

$$([D_{12}]\mathbf{a}_1 - \mathbf{a}_0)^{\mathrm{T}}([D_{12}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \tag{4}$$

$$([D_{13}]\mathbf{a}_1 - \mathbf{a}_0)^{\mathrm{T}}([D_{13}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \tag{5}$$

$$([D_{14}]\mathbf{a}_1 - \mathbf{a}_0)^{\mathrm{T}}([D_{14}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0 \tag{6}$$

$$([D_{56}]\mathbf{a}_{1n} - \mathbf{a}_0)^{\mathrm{T}}([D_{56}]\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0 \tag{7}$$

$$([D_{57}]\mathbf{a}_{1n} - \mathbf{a}_0)^{\mathrm{T}}([D_{57}]\mathbf{a}_{1n} - \mathbf{a}_0) - R_1^2 = 0 \tag{8}$$

Eqs. (9)–(13) were used to calculate five of the six unknowns in $\mathbf{b}_0$, $\mathbf{b}_1$ and $\mathbf{b}_{1n}$. The variable b_{0x} and the link lengths R_1 and R_2 are specified.

$$([D_{12}]\mathbf{b}_1 - \mathbf{b}_0)^{\mathrm{T}}([D_{12}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \tag{9}$$

$$([D_{13}]\mathbf{b}_1 - \mathbf{b}_0)^{\mathrm{T}}([D_{13}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \tag{10}$$

$$([D_{14}]\mathbf{b}_1 - \mathbf{b}_0)^{\mathrm{T}}([D_{14}]\mathbf{b}_1 - \mathbf{b}_0) - R_1^2 = 0 \tag{11}$$

$$([D_{56}]\mathbf{b}_{1n} - \mathbf{b}_0)^{\mathrm{T}}([D_{56}]\mathbf{b}_{1n} - \mathbf{b}_0) - R_1^2 = 0 \tag{12}$$

$$([D_{57}]\mathbf{b}_{1n} - \mathbf{b}_0)^{\mathrm{T}}([D_{57}]\mathbf{b}_{1n} - \mathbf{b}_0) - R_1^2 = 0 \tag{13}$$

3. Trajectory generation

After incorporating the multi-phase motion generation method described in the previous section, the user can synthesize a planar four-bar motion generator and determine the paths of its moving pivots. The moving pivot paths of the follower link must be connected in a manner that will allow smooth displacement, velocity, acceleration and jerk transitions between the determined moving pivot paths. Abrupt or discontinuous transitions will ultimately result in excessive wear on the slider-crank mechanism. The slider path of the equivalent slider-crank motion generator will be comprised of the moving pivot paths of the follower link and the trajectories that connect them.

During the operation of an adjustable four-bar mechanism within a particular phase, the radial positions of the moving pivots of the crank and follower links are constant and the radial velocities, accelerations and jerks of these moving pivots are zero. The same holds true during moving pivot, constant link length adjustments of the adjustable planar four-bar mechanism. When moving pivot *and* link length adjustments considered, the radial positions, velocities, accelerations and jerks of the moving pivots undergo a transition from the link parameters in the former phase to the link parameters in the latter phase. If transition curves are generated for the follower link, and these curves are connected piecewise to the moving pivot curves of the follower corresponding to the phases before and after this transition, a single slider path is generated that will account for the transition between phases (or follower link moving pivot adjustment).

A 7th order polynomial [22,23] is required to specify the radial position, velocity, acceleration and jerk parameters of the moving pivot of the follower link of the adjustable planar four-bar motion generator during the transition between phases.

$$R(\theta) = a_0 + a_1\theta + a_2\theta^2 + a_3\theta^3 + a_4\theta^4 + a_5\theta^5 + a_6\theta^6 + a_7\theta^7 \tag{14}$$

The radial displacement, velocity, acceleration and jerk boundary conditions for this polynomial are

$$R(\theta_0) = R_0 \tag{15}$$

$$\frac{\mathrm{d}R(\theta_0)}{\mathrm{d}\theta_0} = \dot{R}_0 \tag{16}$$

$$\frac{\mathrm{d}R^2(\theta_0)}{\mathrm{d}\theta_0^2} = \ddot{R}_0 \tag{17}$$

$$\frac{\mathrm{d}R^3(\theta_0)}{\mathrm{d}\theta_0^3} = \dddot{R}_0 \tag{18}$$

$$R(\theta_f) = R_f \tag{19}$$

$$\frac{\mathrm{d}R(\theta_f)}{\mathrm{d}\theta_f} = \dot{R}_f \tag{20}$$

$$\frac{\mathrm{d}R^2(\theta_f)}{\mathrm{d}\theta_f^2} = \ddot{R}_f \tag{21}$$

$$\frac{\mathrm{d}R^3(\theta_f)}{\mathrm{d}\theta_f^3} = \dddot{R}_f \tag{22}$$

In this work, the term R_0 is the length of the follower link in phase one (link $\mathbf{b}_0$–$\mathbf{b}_1$) and the term R_f is the length of the follower link in phase two (link $\mathbf{b}_0$–$\mathbf{b}_{1n}$). The constraints specify a linear set of eight equations with eight unknowns whose solutions are

$$a_0 = R_0 \tag{23}$$

$$a_1 = \dot{R}_0 \tag{24}$$

$$a_2 = \frac{\ddot{R}_0}{2} \tag{25}$$

$$a_3 = \frac{\dddot{R}_0}{6} \tag{26}$$

$$a_4 = \frac{35}{\theta_f^4}\left(R_f - R_0 - \dot{R}_0\theta_f - \frac{1}{2}\ddot{R}_0\theta_f^2 - \frac{1}{6}\dddot{R}_0\theta_f^3\right) - \frac{15}{\theta_f^3}\left(-\dot{R}_0 - \ddot{R}_0\theta_f - \frac{1}{2}\dddot{R}_0\theta_f^2\right) + \frac{5}{2\theta_f^2}(-\ddot{R}_0 - \dddot{R}_0\theta_f) + \frac{1}{6\theta_f}\dddot{R}_0 \tag{27}$$

$$a_5 = \frac{-84}{\theta_f^5}\left(R_f - R_0 - \dot{R}_0\theta_f - \frac{1}{2}\ddot{R}_0\theta_f^2 - \frac{1}{6}\dddot{R}_0\theta_f^3\right) + \frac{39}{\theta_f^4}\left(-\dot{R}_0 - \ddot{R}_0\theta_f - \frac{1}{2}\dddot{R}_0\theta_f^2\right) - \frac{7}{2\theta_f^3}(-\ddot{R}_0 - \dddot{R}_0\theta_f) + \frac{1}{2\theta_f^2}\dddot{R}_0 \tag{28}$$

$$a_6 = \frac{70}{\theta_f^6}\left(R_f - R_0 - \dot{R}_0\theta_f - \frac{1}{2}\ddot{R}_0\theta_f^2 - \frac{1}{6}\dddot{R}_0\theta_f^3\right) + \frac{34}{\theta_f^5}\left(-\dot{R}_0 - \ddot{R}_0\theta_f - \frac{1}{2}\dddot{R}_0\theta_f^2\right) + \frac{13}{2\theta_f^4}(-\ddot{R}_0 - \dddot{R}_0\theta_f) + \frac{1}{2\theta_f^3}\dddot{R}_0 \tag{29}$$

$$a_7 = \frac{-20}{\theta_f^7}\left(R_f - R_0 - \dot{R}_0\theta_f - \frac{1}{2}\ddot{R}_0\theta_f^2 - \frac{1}{6}\dddot{R}_0\theta_f^3\right) + \frac{10}{\theta_f^6}\left(-\dot{R}_0 - \ddot{R}_0\theta_f - \frac{1}{2}\dddot{R}_0\theta_f^2\right) - \frac{2}{\theta_f^5}(-\ddot{R}_0 - \dddot{R}_0\theta_f) + \frac{1}{6\theta_f^4}\dddot{R}_0 \tag{30}$$

4. Example problem

Two-phase moving pivot adjustments of the adjustable planar four-bar motion generator with fixed crank and adjustable follower lengths are exemplified in this section. Listed in Table 2 are the X- and Y-coordinates of **p**, **q** and **r** for seven prescribed rigid body positions.

Table 2
Prescribed rigid body positions for the adjustable planar four-bar motion generator

	p	**q**	**r**
Phase 1			
Position 1	−0.5175, 0.9640	−0.2148, 1.5049	0.3551, 1.3103
Position 2	−0.4502, 1.0207	−0.1413, 1.5581	0.4263, 1.3570
Position 3	−0.3786, 1.0720	−0.0645, 1.6064	0.5011, 1.3997
Position 4	−0.3030, 1.1173	0.0152, 1.6492	0.5792, 1.4382
Phase 2			
Position 5	−0.0583, 1.2042	0.4515, 1.6834	0.9782, 1.3914
Position 6	0.1449, 1.2155	0.5383, 1.6945	1.0648, 1.4023
Position 7	0.2319, 1.2195	0.6249, 1.6988	1.1517, 1.4070

Eqs. (4)–(8) were used to calculate five of the six unknowns in $\mathbf{a}_0$, $\mathbf{a}_1$ and $\mathbf{a}_{1n}$. The variable a_{0x} and the link length R_1 were specified ($a_{0x} = 0$ and $R_1 = 1$). Using the following initial guesses:

$$a_{0y} = 0.1 \quad \mathbf{a}_1 = (-0.5, 0.5) \quad \mathbf{a}_{1n} = (-0.5, 0.5)$$

the planar four-bar mechanism solutions converged to

$$a_{0y} = 0.0761 \quad \mathbf{a}_1 = (-0.7049, 0.7859) \quad \mathbf{a}_{1n} = (-0.1739, 1.0608).$$

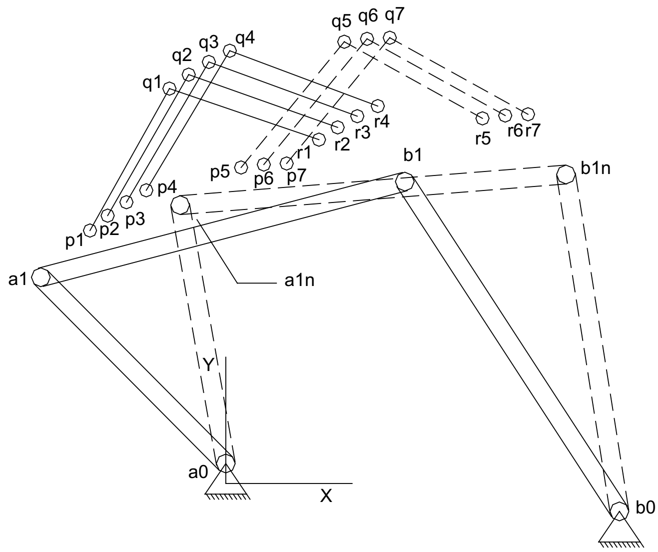

Fig. 7. Adjustable planar four-bar motion generator and corresponding prescribed rigid body positions.

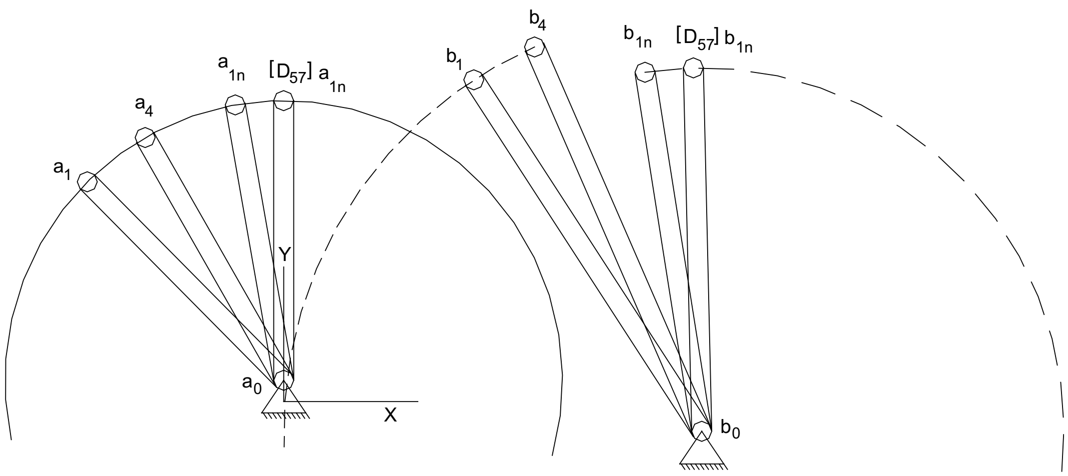

Fig. 8. Moving pivot paths of the synthesized adjustable planar four-bar motion generator.

Eqs. (9)–(13) were used to calculate five of the six unknowns in $\mathbf{b}_0$, $\mathbf{b}_1$ and $\mathbf{b}_{1n}$. The variable b_{0x} and the links length R_1 and R_2 were specified (b_{0x} = 1.5, R_1 = 1.5 and R_2 = 1.3).

Using the following initial guesses:

$$b_{0y} = 0.1 \quad \mathbf{b}_1 = (0.6, 1.2) \quad \mathbf{b}_{1n} = (0.6, 1.2)$$

the planar four-bar mechanism solutions converged to

$$b_{0y} = -0.1064 \quad \mathbf{b}_1 = (0.6821, 1.1505) \quad \mathbf{b}_{1n} = (1.2964, 1.1775).$$

Using the calculated fixed and moving pivot parameters, the resulting adjustable planar four-bar motion generator is illustrated in Fig. 7. An equivalent slider-crank motion generator was designed for the planar four-bar motion generator in this work.

In Fig. 8, the starting and ending positions for the synthesized adjustable planar four-bar motion generator in phases one and two are illustrated. Since the crank link underwent a constant

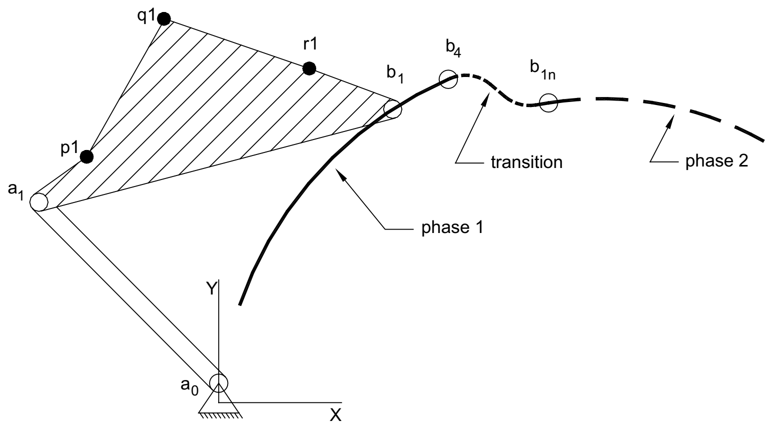

Fig. 9. Equivalent slider-crank motion generator and initial rigid body position.

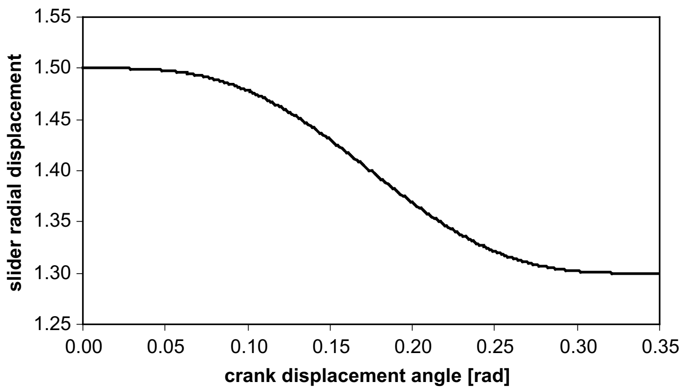

Fig. 10. Radial slider displacement versus crank angle for synthesized slider-crank motion generator.

link length moving pivot adjustment, all of the moving pivot positions ($\mathbf{a}_1$ through $\mathbf{a}_4$ and $\mathbf{a}_{1n}$ through $[D_{57}]\mathbf{a}_{1n}$) for this link lie on the same circle. The moving pivot positions ($\mathbf{b}_1$ through $\mathbf{b}_4$ and $\mathbf{b}_{1n}$ through $[D_{57}]\mathbf{b}_{1n}$) for the follower link lie on two different circles (one for each phase). To complete the slider path of the equivalent slider-crank motion generator (the path between $\mathbf{b}_1$ and $\mathbf{b}_{1n}$), Eq. (14) was used calculate a path to link both of the follower moving pivot arcs in Fig. 8. Using Eq. (14) and the prescribed boundary conditions, the slider path in Fig. 9 was designed. This slider path produces the radial displacement, velocity, acceleration and jerk profiles illustrated in the Figs. 10–13.

Listed in Table 3 are the X- and Y-coordinates of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ for seven prescribed rigid body positions generated by the equivalent planar slider-crank mechanism. To achieve positions 2, 3 and 4 in Table 3, link $\mathbf{a}_0$–$\mathbf{a}_1$ is rotated to 130°, 125° and 120° with respect to the X-axis. To achieve positions 5, 6 and 7 in Table 3, link $\mathbf{a}_0$–$\mathbf{a}_{1n}$ must rotated to 100°, 95° and 90° with respect to the X-axis. In both phases, the crank angle is initially 135° with respect to the X-axis and the rigid body point coordinates at this crank position are the coordinates in position 1 of Table 3.

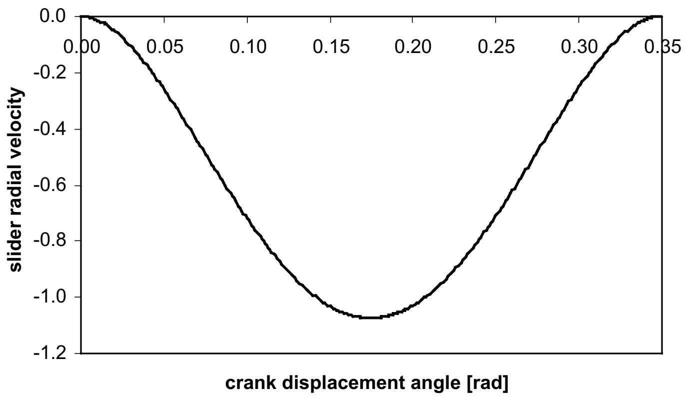

Fig. 11. Radial slider velocity versus crank angle for synthesized slider-crank motion generator.

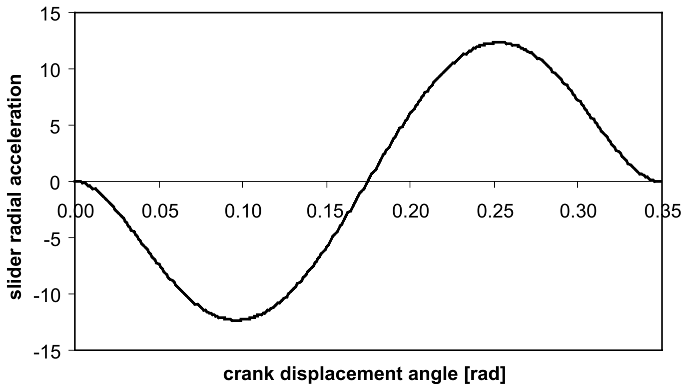

Fig. 12. Radial slider acceleration versus crank angle for synthesized slider-crank motion generator.

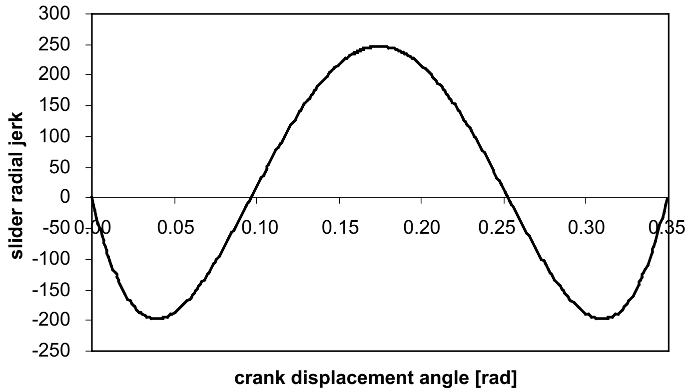

Fig. 13. Radial slider jerk versus crank angle for synthesized slider-crank motion generator.

Table 3
Rigid body positions generated by equivalent slider-crank motion generator

	p	q	r
Phase 1			
Position 1	−0.5175, 0.9640	−0.2148, 1.5049	0.3551, 1.3103
Position 2	−0.4508, 1.0205	−0.1417, 1.5577	0.4258, 1.3564
Position 3	−0.3797, 1.0715	−0.0654, 1.6057	0.5002, 1.3988
Position 4	−0.3046, 1.1167	0.0138, 1.6485	0.5778, 1.4372
Phase 2			
Position 5	0.0794, 1.2084	0.4747, 1.6858	1.0001, 1.3915
Position 6	0.1493, 1.2578	0.5510, 1.7298	1.0724, 1.4285
Position 7	0.2232, 1.3029	0.6302, 1.7704	1.1482, 1.4632

Illustrated in Figs. 10–13 are the radial displacement, velocity, acceleration and jerk profiles (with respect to crank displacement angles) during the transition from phase 1 to phase 2 for the equivalent slider-crank motion generator. The radial velocity, acceleration and jerk profile boundary conditions (Eqs. (16)–(18) and Eqs. (20)–(22)) were specified to zero in order to generate velocity, acceleration and jerk profiles that are continuous with those profiles outside of the transition. The radial displacement profile boundary conditions (Eqs. (15) and (19)) were specified to the phase 1 and phase 2 lengths of the follower link ($R_0 = 1.5$ and $R_f = 1.3$) in order to generate displacement profiles that are continuous with those profiles outside of the transition also.

5. Discussion

The slider path design method presented in this work is applicable for most of the existing motion generation methods for adjustable planar four-bar mechanisms since only the fixed and moving pivots for the synthesized mechanism are required. Although a two-phase moving pivot problem was exemplified in this work, additional phases of prescribed rigid body positions can be incorporated. By calculating the fixed and moving pivots of the planar four-bar mechanism for each additional phase, and additional transition paths for each additional phase (using Eqs. (14)–(30)) the equivalent slider-crank motion generator can be designed to achieve additional phases. Although the position of a rigid body in two-dimensional space is commonly described by a single point and a displacement angle (**p** and θ for example), the authors chose to describe the rigid body using three points for computational purposes. If the user prefers to describe the rigid body using conventional notation, the displacement matrix in Eq. (2) will be replaced with the conventional plane rigid body displacement matrix [20,21]. Computer aided design software was to prescribe the mechanism parameters in this work and mathematics software was used to compute the mechanism solutions. This software enabled the tabulated prescribed and calculated parameters to be expressed in four significant figures.

6. Conclusion

A design method for slider-crank mechanisms to achieve multi-phase motion generation applications accomplished by planar four-bar mechanisms with adjustable moving pivots is presented in this work. The benefit of the method is that using it, multiple phases of prescribed rigid body positions are achievable using a mechanism with fewer moving parts than the planar four-bar mechanism. Another benefit of this method is that using it, slider-crank mechanisms can be designed to achieve phases of prescribed rigid body positions without any physical or automated adjustments of its moving pivots between phases. A slider path that enables the slider-crank mechanism to achieve two phases of prescribed rigid body positions is designed by using 7th order polynomials to connect the moving pivot paths of the follower link of the adjustable planar four-bar mechanism. The example problem in this work considers a two-phase moving pivot adjustment of the adjustable planar four-bar mechanism.

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