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Optmization of Parallel Mechanisms with Two or Three Degrees of Freedom

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Abstract: Parallel manipulators for the machine tool industry have been studied extensively for various industrial applications. However, limited useful workspace areas, the poo rmobility, and design difficulties of more complex parallel manipulators have led to more interest in parallel manipulators with less than six degrees of freedom(DoFs). Several parallel mechanisms with various numbers and types of degrees of freedom are described in this paper, which can be used in parallel kinematics machines, motion simulators and industrial robots.

Keywords: parallel manipulator parallel kinematic machine degree of freedom robot

Introduction

Mechanical systems that allow a rigid body to move with respect to a fixed base play a very important role in numerous applications. A rigid body can move in various translational or rotational directions which a recalled degrees of freedom(DoFs). The total number of degrees of freedom for a rigid body can not exceed six , for example, three translation motions along mutually or thogonal axes and three rotational motions about these axes. A robot includes a system to control several degrees of freedom of an end effector.

The last few years have witnessed important developments in the use of industrial robots, mainly due to their flexibility. However, the mechanical architecture of the most common robot is not well adapted to certain tasks. Other types of architectures have, therefore, recently been developed for industrial use, including parallel manipulators.

A parallel manipulator, which is a closed loop several limbs or legs. Typically, the number of limbs is equal to the number of degrees of freedom such that every limb is controlled by one actuator and all the actuators can be mounted at or near the fixed base. For this reason , parallel manipulatorsare sometimes called platform manipulators. Because the external load can be shared by the actuators, parallel manipulators tend to have a large load carrying capacity.Parallel manipulators are always presented as having very good performance in terms of accuracy, rigidity and the ability to manipulate large loads.They have been used in a large number of applications ranging from a stronomy to flight simulators, and are becoming increasingly popular in the machine tool industry.

The conceptual design of parallel manipulators can be dated back to 1947,when Goughe stablished the basic principles of a mechanism with a closed-loop kinematic structure[1] to control the position and orientation of a moving platform to test tire wear and damage. He built a prototype in 1955[2] (Fig. 1a) where the moving element was a hexagonal platform whose vertices were all connected to links by ball and socket joints. The other end of the link was attached to the base by auniversal joint. Six linear actuators modified the total link length. Stewart designed a platform manipulator as an aircraft simulator in 1965[3]. (Fig.1b) , in which the moving element was a triangular platform whose vertices were all connected by ball-and- socket joints to support mechanisms each constituting of two jacks, also placed in a triangle. In 1978, Hunt[4] made a systematic study of the kinematic structure of parallel manipulators, with the planar three RPS parallel manipulator as a typical example. Since then , parallel manipulators have been studied extensively by numerous researchers.

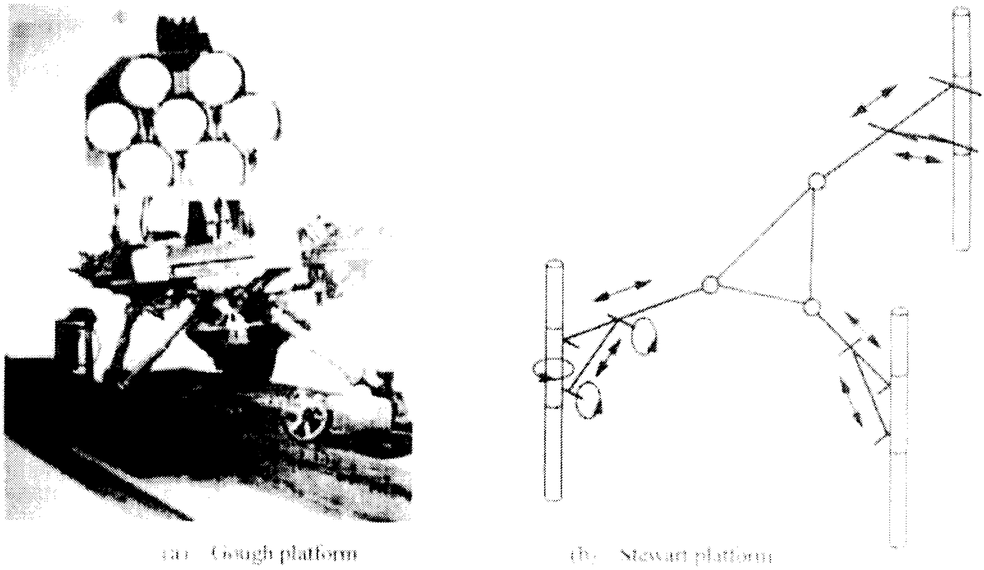

(a) Gough platform (b) Stewart platform

Fig. 1 Early designs of parallel manipulators

Most of the six DoFs parallel manipulators studied to date have included six extendible limbs. These parallel manipulators possess the advantages of high stiffness,low inertia,and large pay load capacity. However, they suffer the problems of relatively small useful workspace and design difficulties.Furthermore,their direct kinematics is very difficult to analyze. Therefore, parallel manipulators with less than six DoFs have increasingly attracted attention for industry applications.

This paper introduces parallel manipulators and the classification of parallel manipulators. Three types of new parallel manipulators are introduced: a spatial three DoFs parallel manipulator, a two DoFs parallel manipulator , and aplanar three DoFs serial parallel manipulator.

1 Definition of a Parallel Manipulator

A parallel manipulator is made of an end effector with ***n*** degrees of freedom with a fixed base linked together by at least two independent kinematic linkages. Actuation takes place through simple actuators[5].

These mechanisms have the following number of degrees of freedom of then end effector:

- At least two linkages support the end-effector. Each of those linkages contains at least one simple actuator.
- The number of actuators is the same as the number of degrees of freedom of the end-effector.
- The mobility of the manipulator is zero when the actuators are locked.

Parallel mechanisms are of interest for the following reasons:

- The load can be distributed on the multiple linkages.
- Few actuators are needed.
- When the actuators are locked, the manipulator remains in position , which is an important safety concern for certain applications.

Parallel manipulators for which the number of linkages is strictly equal to the number of degrees of freedom of the end effector are called fully parallel manipulators.

2 Degrees of Freedom of a Mechanism

The degrees of freedom of a mechanism are the number of independent parameters or inputs needed to completely specify the configuration of the mechanism. However, a general mobility criterion can not be easily defined for closed loop kinematic linkages, as Hunt[4] and Lerbet[6] already noted. Classical mobility formulae can indeed neglect some degrees of freedom. Grubler's formula[7] is nevertheless generally used, which may be written as:

$$M = d(n - g - 1) + \sum_{i=1}^{g} f_i \qquad (1)$$

Where ***M*** is system mobility (degrees of f reedom) ;**d** is screw system order (**d** =**3** for planar and spherical motion , **d** = **6** for spatial motion) ; **n** is number of links including the frame; **g** is number of joints; and $\mathbf{f_i}$ are degrees of freedom associated with the **i**-th joint.

3 Classification of Parallel Manipulators

The total number of degrees of freedom of a rigid body can not exceed 6; therefore, the number of DoFs of a parallel manipulator will be between 2 and 6.Since the first parallel mechanism design ,many mechanical designs have been proposed for parallel manipulators with 2 to 6 DoFs. A survey of 87 actuators proposed in the literature showed that 40% has six DoFs, 3.5% five DoFs, 6% four DoFs,40% three DoFs,and the remaining two DoFs [7].

3.1 Two-DoFs parallel manipulators

Most existing two DoFs parallel manipulators are planar manipulators with two translational DoFs. Such designs use only prismatic and revolute joints.McCloy[8] showed that there are 20 different combinations.This number is reduced to 6 as shown in Fig.2 if the actuators are assumed to be attached to the ground. There is no passive prismatic joint and no actuator is supporting the weight of another actuator.

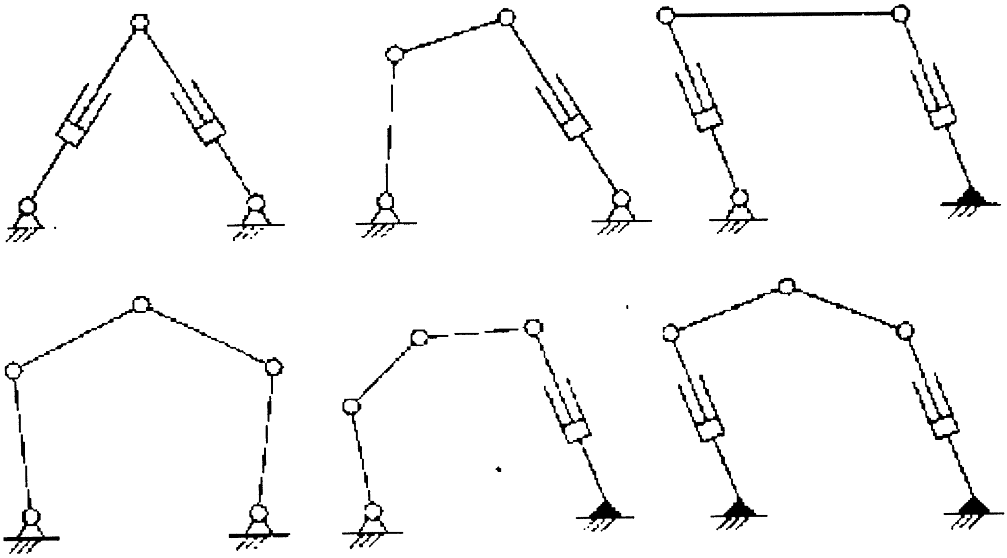
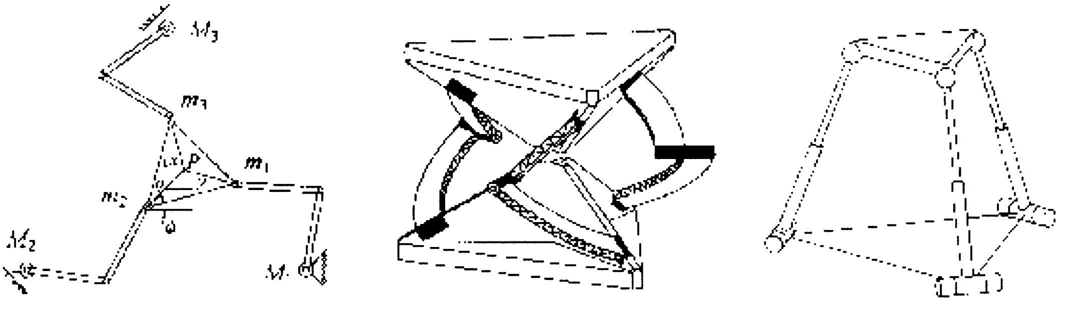


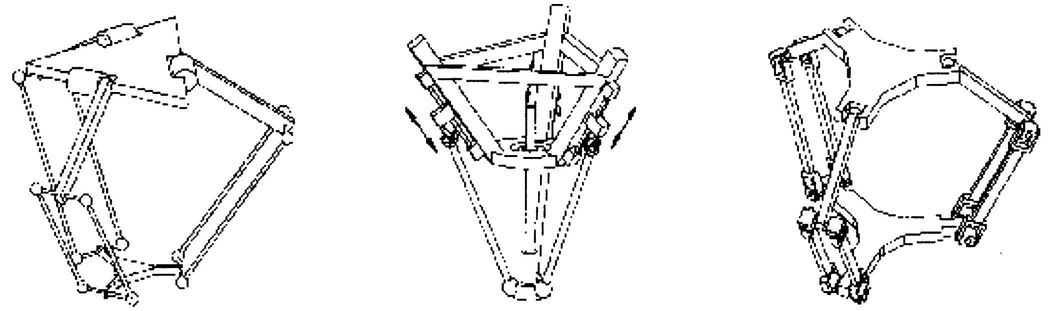
Fig.2 Six types of two-DoFs parallel manipulators

3.2 Three-DoFs parallel manipulators

There are many three DoFs parallel manipulators, soonly the classical designs will be presented here.One example is the planar three ***RRR*** (***R*** stands for revolving joint) parallel manipulator as shown in Fig.3a. The moving platform has three planar DoFs, which are two translations along the x and y axes and one rotation around the axis perpendicular to the ***Oxy*** plane. Another example is the spherical three ***RRR*** parallel manipulator as shown in Fig.3b , in which all the joint axes intersect at a common vertex. The motion of any point in the mechanism is rotation about the vertex. The moving platform has only rotational.prismatic joint and no actuator is suporting the DoFs with respect to the base. Hunt[4] presented the weight of another actuator.three ***RPS*** parallel manipulator shown in Fig.3c,



(a) Planar three-RRR manipulator (b) Spherical three-RRR manipulator (c) Three-RPS manipulator



(d) DELTA robot (e) Manipulator with passive constraints (f) Tsai's manipulator

Fig.3 Some three-DoFs parallel manipulators

which has complex DoFs, which can not be strictly defined. The most famous robot with three translations is the DELTA[9](Fig. 3d) , proposed by Clave[9] and marketed by the Demaurex Company and ABB under the name IRB 340 FpexPicker DELTA has been widely used in industry. Another type of three DoFs parallel manipulator has the moving platform connected to the base through four legs, where the fourth leg is passive and is also the leading leg, which means that the leg determines the motion of the moving platform , for example, in the spherical coordinate parallel manipulator shown in Fig. 3e. This parallel manipulator is used for the machine tool design by IFW of the University of Hannover[10].

3.3 Four-DoFs parallel manipulators

A four DoFs fully parallel manipulator has $d = 6$, $n= 10$, $g = 12$, and $M = 4$. Substuting into Eq.(1) gets F=224, which is the degrees of freedom for each leg. Therefore,there are, actually, no four DoFs fully parallel manipulators. The early mechanisms with four DoFs were not fully parallel manipulators, i.e. , manipulators with two actuators per linkage or with passive constraints.

.3. 4 Five-D oFs parallel manipulators

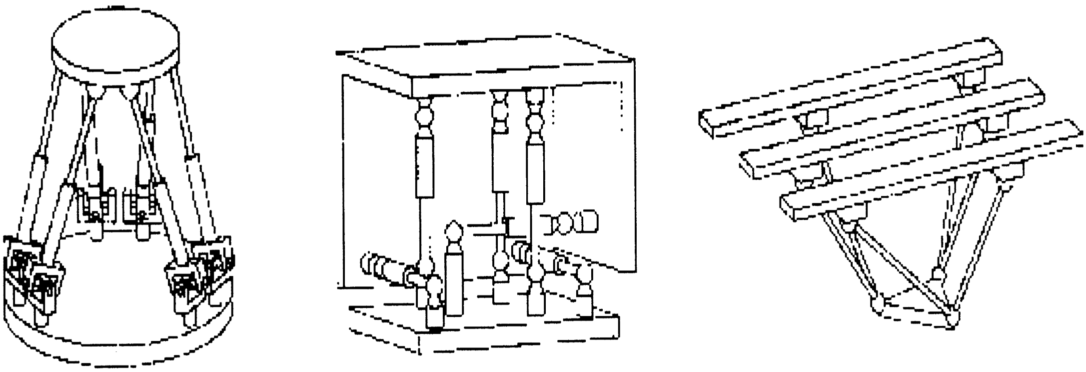
Five DoFs fully parallel manipulators must have F= 295,so there are no five DoFs fully parallel manipulators. A five DoFs parallel manipulator proposed by Austad[11] consists of two parallel manipulators.

3.5 Six-DoFs parallel manipulators

Six DoFs parallel manipulators are the most popular manipulators so they have been studied extensively. The architecture shown in Fig.4a is a classical six DoFs parallel manipulator. Most six DoFs parallel manipulators have six extendible limbs. These parallel manipulators possess the advantages of high stiffness,low inertia, and large pay load capacity.How ever,they suffer the problems of relatively small useful workspace and design difficulties. Furthermore,analysis of their direct kinematics is very difficult. There are also some exotic chain manipulators in which the manipulator is actuated by a planar mechanism , such as a four bar mechanism ,or a five bar mechanism ,or which have two actuators per leg and which usually have three legs.

4 Evolution of Parallel Manipulators

After Gough established the basic principles of mechanisms with closed loop kinematic structures in 1947, as shown in Fig.1a,many other parallel manipulators with a specified number and type of degrees of freedom have also been proposed. The architecture designed by Stewart in 1965 is shown in Fig.1b. As shown in Fig.4a , theoretically speaking,the six legs can be arranged at will to design various six DoFs parallel manipulators, such as the manipulator shown in Fig.4b , where the legs are arranged in a 3-2-1 style which is a very compact structure that can be used in microsystem [12]. The arrangement of the six legs shown in Fig. 4c makes the manipulator move freely along a specified direction , which is very useful for the industrial applications.



(a) General manipulator (b) Manipulator with 3-2-1 arrangement (c) Slide manipulator

Fig.4 Some six-DoFs parallel manipulators

A six DoFs parallel manipulator similar to that proposed by Pierrot[13] has each pair of legs in the manipulator shown in Fig.4a parallel to each other. The number of DoFs of the manipulator will be different if the inputs to the two legs in each pair are the same. The equivalent manipulator architecture is shown in Fig.5.The manipulator output will be three translations, which is probably the origin of the DELTA robot.The actuated links can be arranged as the well known ,fast robot DELTA shown in Fig.3d. DELTA has been made in several versions, such as the Pollard mechanism[14].Tsai's manipulator[15], Fig.3 f, is also among three translational parallel manipulators. Although Tsai's manipulator has translations identical with that of DELTA , it is not exactly a version of DELTA. Their design concepts are different and Tsai's manipulator is the first design to deal with the problem of a UPU chain. Another three translational DoFs parallel manipulator, Star, was design by Hervé[16] based on the group theory.

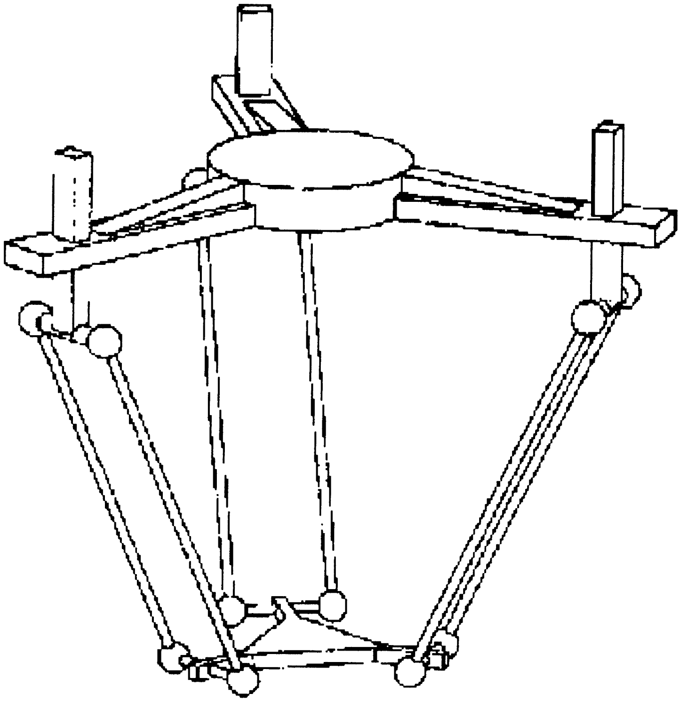


Fig.5 The apology architecture of DELTA

Although these design concepts provide ideas to design a new manipulator, additional work is needed to design a robot combining translational and rotational DoFs with less than six DoFs. For example,there are few spatial three DoFs parallel manipulators combining two spatial translations and one rotation , as will be presented in the following section.

5 New Spatial Three-DoFs ParallelManipulator

5 .1 Manipulator structure

The spatial three DoFs parallel manipulator[17] shown in Fig.6a consists of a base plateform,a movable platform , and three legs that connect the two plates. Each connecting leg has four degrees of freedom.Two of the three legs have identical chains with a two-DoFs joint or two- DoF joints. The third leg consists of a planar four bar parallelogram and three -DoF joints. One-DoF joint in each leg is actuated.

The moving platform is an isosceles triangle.The vertices of the platform are connected to a fixed base plate through legs (1) , (8) and (12).Legs (1) and (12) have identical chains with a constant link connected to a universal joint (or two revolute joints) (15) or (13) at the bottom end and a passive revolute joint (3) or (11) at the other end. The revolute joint is then attached to an active slider (4) or (10) , which is mounted on the guideway (2) or (9). The third leg (8) has a constant link , a planar four bar parallelogram , which is connected to a revolute joint (16) at the bottom end and a passive revolute joint (5) at the other end. The revolute joint is attached to an active slider (6) which is mounted on the guideway (7). The movement of the moving platform is accomplished by sliding the three sliders on the guideways.

5.2 Manipulator capability

The proposed manipulator is a general manipulation device that must have three degrees of freedom when the input elements are active. In the arrangement of the links and joints shown in Fig.6, legs (1) and (12) provide two constraints on the rotation of the moving platform about the z axis and the translation along the x axis. The revolute joints (5) and (16) for the third leg (8) have parallel axes as shown in Fig.6a. The third leg can provide one constraint on the rotation of the moving platform about the axis. Hence, the combination of the three legs constrains the rotation of the moving platform with respect to the x and z axes and the translation along the axis. Therefore, the mechanism has two translational degrees in the ***Oyz*** plane and one rotational degree of freedom about they axis.

5.3 Novelties and applications

The mechanical design is interesting because of the third actuatin leg mechanism which uses a planar four bar parallel ogram ,as used in other parallel mechanisms,such as Star Like robot, the Tsai manipulator, CaPaMan [18], This unique spatial three DoFs parallel manipulator (a) has only revolute joints,(b) combines spatial translational and rotational degrees freedom in a spatial three DoFs parallel manipulator, and (c) has high mobility of the rotational DoF. The design can be practically applied to parallel machine tools. Because of the low mobility and flexibility of six DoFs parallel mechanisms, more and more parallel machine tools are built as hybrid structures, such as those of Tricept and GeorgeV, which are usually based on three DoF sparallel mechanisms. The proposed parallel manipulator will be designed as a hybrid parallel machine tool in future work.The parallel mechanism can also be used as an indust rial robot, a montion simulator, or a micromanipulator.

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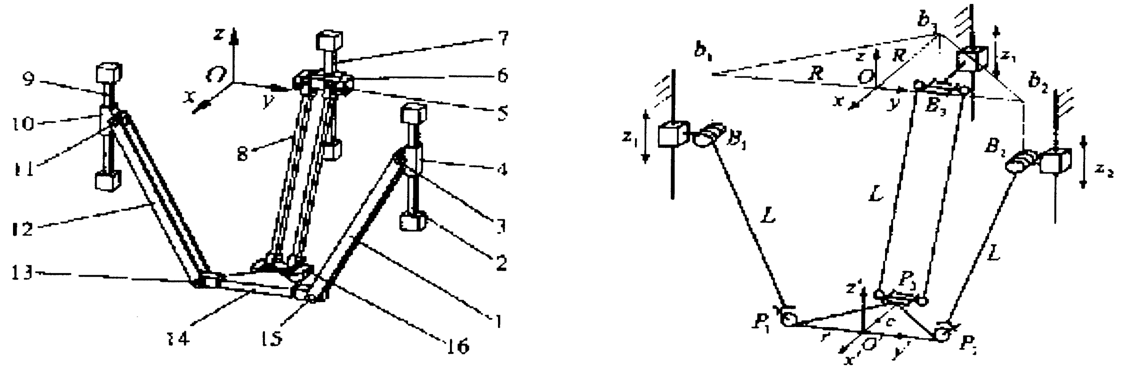


(a) (b)

Fig.6 New spatial three-DoFs parallel manipulator

.The design shown in Fig.6ais over-constrained, which means that the machined components must be accurate. However, the universal joints can be replaced by two revolute joints which are not difficult to fabricate accurately.

5.4 Inverse kinematics problem

The inverse kinematics problem involves mapping a known pose (position and orientation) of the output platform to a set of input joint variables that will achieve that pose. A kinematics mode l of the manipulator is shown in Fig.6b. The output platform vertices are denoted as the platform joints, ***Pi*** (i= 1, 2, 3) , w ith the vert ices of the base platform denoted as **bi** (*i*= 1, 2, 3). A fixed global reference system: **O-xy***z* is located at the center of side **b1b2** with the **z** axis normal to the base plate and the *y* axis directed along **b1b2**. Another reference frame, called top fram′ ***O′*** *-x ′ y′ z′* , is located at the center of side **P 1P 2**. The **z** axis is perpendicular to the output platform and the **y** axis is directed along **P1P2**. The length of each leg link is denoted by *L* , where *P iB i*= *L* , *i*= 1, 2, 3. In some cases, the length of link **P3B3** can differ from that of **P1B1** and ***P2B2***.

The objective of the inverse kinematics solution is to define a mapping from the pose of the output platform in Cartesian space to the set of actuated inputs that achieve that pose. The pose of the moving platform is considered known, with the position given by the position vector O ',

$$O'_{\mathfrak{R}} = (x \quad y \quad z)^{T} \qquad (2)$$

Where x = 0.The orientation is given by a matrixi Q

$$\mathbf{Q} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \qquad (3)$$

Where the angle ϕ is the rotation of the output platform with respect to the y axis. The coordinates of point P_i in the frame can be described by the vector p_i (i= 1, 2, 3) :

$$\begin{aligned} \mathbf{p}_{1\mathfrak{R}} &= (0 \quad -r \quad 0)^{T}, \\ \mathbf{p}_{2\mathfrak{R}} &= (0 \quad r \quad 0)^{T}, \\ \mathbf{p}_{3\mathfrak{R}} &= (-r \quad 0 \quad 0)^{T} \end{aligned} \qquad (4)$$

Vectors $\mathbf{b}_i$ (i = 1, 2, 3) will be defined as the position vectors of base joints in frame,

$$\begin{aligned} \mathbf{b}_{1\mathfrak{R}} &= (0 \quad -R \quad z_1)^{T}, \\ \mathbf{b}_{2\mathfrak{R}} &= (0 \quad R \quad z_2)^{T}, \\ \mathbf{b}_{3\mathfrak{R}} &= (-R \quad 0 \quad z_3)^{T} \end{aligned} \qquad (5)$$

The vector $\mathbf{p}_i$ (i= 1, 2, 3) in frame **O-xyz** can be written as

$$\mathbf{p}_{i\mathfrak{R}} = \mathbf{Q}\mathbf{p}_{i\mathfrak{R}} + \mathbf{O}'_{\mathfrak{R}} \qquad (6)$$

Then the inverse kinematic of the parallel manipulator can be solved by the following constraint equation ,

$$\| \mathbf{p}_{i\mathfrak{R}} - \mathbf{b}_{i\mathfrak{R}} \| = L, \quad i = 1, 2, 3 \qquad (7)$$

Hence,for a given manipulator and for prescribed values of the position and orientation of the platform, the required actuator inputs can be directly computed from Eq (7)

$$z_1 = \pm\sqrt{L^2 - (-r + y + R)^2} + z \qquad (8)$$

$$z_2 = \pm\sqrt{L^2 - (r + y - R)^2} + z \qquad (9)$$

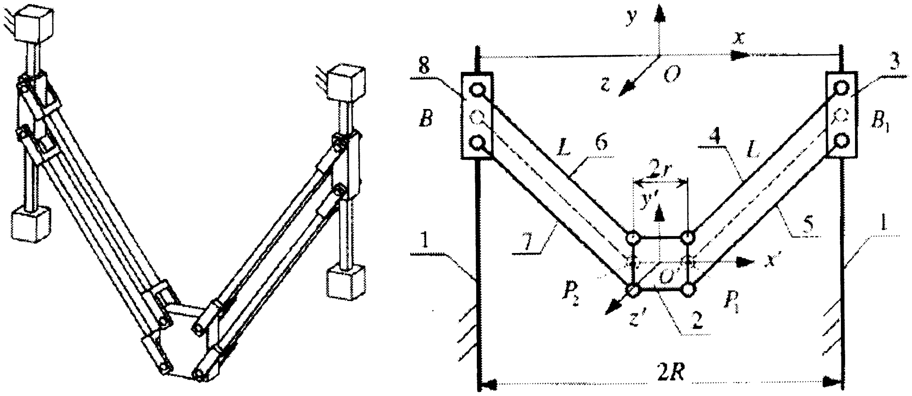
$$z_3 = \pm\sqrt{L^2 - (-r\cos\phi + R)^2 - y^2} + r\sin\phi + z \qquad (10)$$

From Eqs. (8) - (10) ,we can see that there are eight inverse kinematic solutions for a given pose of the parallel manipulator.configuration shown in Fig. 6, each one of the signs“±”in Eqs. (8) - (10) should be“+ ”.

6 Other Parallel Architectures

6.1 Novel two-DoFs translational platform

A novel two DoFs parallel mechanisms [19] is shown in Fig.7a. A schematic of the mechanism is shown in Fig. 7b , where the base is labeled 1 and the moving platform is labeled 2 . The moving platform is connected to the base by two identical legs . Each leg consists of a planar four bar parallelogram : links 2, 3, 4, and 5 for the first leg; 2, 6, 7, and 8 for the second leg.The joints in each planar four bar parallelogram are all revolute pairs.Links 3 and 8 are actuated by prismatic actuators.The platform motion is achieved by the movements of links 3 and 8 transmitted to the platform by the two parallelograms.The moving platform has two pure translational degrees of freedom with respect to the base because of the planar four bar parallelograms. The system is over constrained since only one planar four bar parallelogram is needed to obtain two DoFs of a rigid body in this design. The two planar four bar parallelograms are used to increase the system’s stiffness and to make the system symmetric. This mechanism is now being used to develop a new type of five axis machine tool in cooperation with the Second Machine Tool Factory in Qiqihaer, China.



(a) (b)

Fig 7 Novel planar two-DoFs parallel manipulator

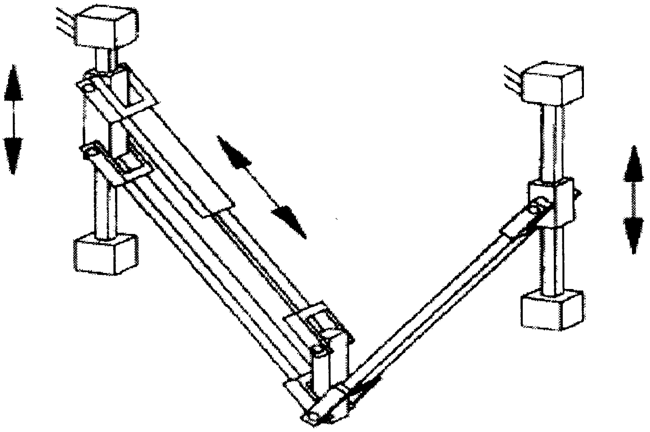


Fig 8 Three-DoFs planar serial-parallel mechanism

6.2 Three-DoFs planar serial- parallel mechanism .

The mechanism [20] shown in Fig.8 has a moving platform connected to the base by two legs.The first leg consists of a constant link which is connected to a revolute joint at the bottom end and a passive revolute joint at the other end. The revolute joint is attached to the base through a prismatic joint.The second leg is very different from the first one. Four sides of the quadrangle are linked to each other through revolute joints. The quadrangle is connected to the base by a prismatic joint. This mechanism has been used to build a new type of four axis machine tool in cooperation with the Jiangdong Machine Tool Factory.

7 Conclusions

This paper analyzes the definition of a parallel manipulator and the degrees of freedom of a mechanism.Three types of new parallel manipulator, a two DoFs parallel manipulator,and a planar three DoFs serial parallel manipulator, are discussed. Two of these designs are being applied to new industrial machines.

8 Acknowledgements

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